Chaos in Financial Markets

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November 2008

Abstract

This paper reviews the literature on chaos in financial time series, and concludes that there is little evidence of low-dimensional chaos in financial markets.

1 Chaos

Chaos has a precise meaning within the world of physics and non-linear mathematics, but applications of 'chaos theory' in other domains (in management, for example) are generally bogus. Mathematical definitions of chaos vary, what follows is an informal one.

Chaos exists when a deterministic dynamical system is sensitive to initial conditions and gives rise to effectively unpredictable long-term behaviour.

Note that high dimensional chaos is indistinguishable from a stochastic process, we're interested in whether markets exhibit low-dimensional chaos.

2 Literature Review

- Savit (1988) wrote an introduction to chaos in market prices.
- Frank and Stengos (1989) examined gold and silver returns and found that the correlation dimension is between 6 and 7 while the Kolmogorov entropy is about 0.2 for both assets.
- Peters (1991) claimed to have found chaos in the financial markets.
- Brock, Hsieh and LeBaron (1991) concluded that the evidence for the presence of deterministic low-dimensional chaotic generators in economic and financial data is not very strong.
- Blank (1991) analysed the futures prices for the S&P 500 index and soybeans. All of their results were consistent with those of markets with underlying generating systems characterized by deterministic chaos (they

give necessary, but not sufficient, conditions to prove the existence of deterministic chaos).

- In an excellent paper, Hsieh (1991) found no evidence of low complexity chaotic behaviour in stock returns.
- Willey (1992) tested the daily prices of the S&P 100 and the NASDAQ-100. Deterministic chaos was rejected by two of three recently developed empirical tests.
- Decoster, Labys and Mitchell (1992) searched for evidence of chaos in commodity futures (silver, copper, sugar and coffee) prices and found evidence of non-linear structure. Evidence for the presence of chaos is provided, but, further research is needed before they can confirm or reject the discovery of chaos.
- Mayfield and Mizrach (1992) estimate the dimension of the S&P 500 (sampled at approximately 20-second intervals) and conclude that the data are either of low dimension with high entropy or non-linear but of high dimension.
- Yang and Brorsen (1993) found evidence of non-linearity in several futures markets, which was consistent with deterministic chaos in about half of the cases.
- Abhyankar, Copeland and Wong (1995) tested for the presence of chaos in the FTSE 100 Index using a six month sample of about 60,000 minute-by-minute returns and found little to support the view that the process is chaotic at any frequency.
- Sewell, *et al.* (1996) examined weekly changes for the period 1980 to 1994 in six major stock indices (the US, Korea, Taiwan, Japan, Singapore and Hong Kong) and the World Index as well as the corresponding foreign exchange rates between the US and the other five countries. They concluded that '[t]hese results do not prove the existence of chaos in these markets but are consistent with its existence in some cases.'
- Abhyankar, Copeland and Wong (1997) tested the world's four most important stock market indices: the S&P 500, the DAX, the Nikkei 225 and the FTSE 100 Index and found no evidence of low-dimensional chaotic processes.
- Serletis and Gogas (1997) tested for deterministic chaos in seven East European black market exchange rates and concluded that there is evidence consistent with a chaotic non-linear generation process in only two out of seven series.
- Barkoulas and Travlos (1998) investigated the existence of a deterministic non-linear structure in the stock returns of the Athens Stock Exchange (an emerging capital market) and found no strong evidence of chaos.

- In a working paper, Wei and Leuthold (1998) looked at six agricultural futures markets—corn, soybeans, wheat, hogs, coffee and sugar—and found that five of them (all except sugar) were chaotic processes.
- Gao and Wang (1999) examined the daily prices of four futures contracts (S&P 500, JPY, DEM and Eurodollar) and found no evidence of deterministic chaos.
- Andreou, Pavlides and Karytinos (2000) examined four major currencies against GRD and found evidence of chaos in two out of four.
- Panas and Ninni (2000) found strong evidence of chaos in daily oil products for the Rotterdam and Mediterranean petroleum markets.
- Adrangi, *et al.* (2001) tested for the presence of low-dimensional chaotic structure in crude oil, heating oil and unleaded gasoline futures prices from the early 1980s and found no evidence of chaos.
- Antoniou and Vorlow (2005) investigated the 'compass rose' patterns revealed in phase portraits (delay plots) of FTSE 100 stock returns and found a strong non-linear and possibly deterministic signature in the data-generating processes.

3 Conclusion

There is little evidence of low-dimensional chaos in financial markets.

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