

# Decision Making Under Risk: A Prescriptive Approach

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Behavioral Finance & Economics Research Symposium - 2009  
Chicago

23–25 September 2009

## Abstract

This article takes the form of a research proposal. The paper seeks to address the problem faced by investors of how best to deal with decision making under risk. The methodology utilizes a *normative* model from economics—expected utility theory—and a *descriptive* model from psychology—prospect theory—to formulate a rational but realistic compromise between the two: a *prescriptive* model of risk preferences. The paper recommends implementing a generic risk metric and a questionnaire for formulating personal risk profiles. The research should be of great interest to both academia and the financial industry.

# 1 Research Problem

From the field of economics, expected utility theory is a *normative* model of decision making under risk. In contrast, from the field of psychology, prospect theory is a *descriptive* model of decision making under risk.

An investment manager is responsible for (either explicitly or implicitly) implementing an algorithm that employs both rational and realistic risk preferences. We define such a compromise between a normative model and a descriptive model as a *prescriptive* model. The problem then is how to build a prescriptive model of decision making under risk.

Although expected utility theory generally assumes that individuals are risk averse, it does not dictate to what degree. Utilizing prospect theory to determine risk aversion in expected utility theory is not straightforward because prospect theory violates expected utility theory.

The problem is exacerbated if one's risk tolerance is context dependent. As John Maynard Keynes quipped, 'there is nothing so dangerous as the pursuit of a rational investment policy in an irrational world'.

More generally, if risk is a personal thing, how should we address global risks? Further, in order to assess global risks such as climate change, we must also incorporate the concept of time. When dealing with social, rather than monetary issues, how should we discount the future?

# 2 Rationale

The research problem is important because it is of great interest to the investment industry. The almost ubiquitous normative performance metric, the Sharpe ratio, is flawed because it ignores higher moments. The most celebrated descriptive model of behaviour under uncertainty, prospect theory, models the fact that people can be risk seeking, which is not a rational investment strategy. The need for an effective compromise is paramount. More generally, the concept of augmenting the usual dichotomy of normative ('ought') and descriptive ('is') with a prescriptive solution (an effective compromise between the two) is of intellectual interest.

# 3 Literature Review

First, we must decide how best to deal with uncertainty. A Dutch book is a gambling term for a set of odds and bets which guarantees a profit, regardless of the outcome of the gamble. At the very least, one who practices self-consistent reasoning should not be susceptible to having a Dutch book made against them. If an individual is not susceptible to a Dutch book, their previsions are said to be *coherent*. A set of betting quotients is coherent if (Ramsey 1926; de Finetti 1937; Shimony 1955) and only if (Kemeny 1955; Lehman 1955) they satisfy the axioms of probability. On this basis, it is my view that probability is both necessary and sufficient when dealing with uncertainty.

What sort of innate risk preferences are likely to have evolved? More specifically, why are we generally risk averse? In order to propagate our genes, we need to survive. By definition, I have survived thus far, everything that I have already experienced can not be fatal, because I am alive. I have never eaten that berry before, and I have survived, so why should I risk eating it? The most fundamental bias, therefore, is the status quo bias (also known as conservatism). Sinn and Weichenrieder (1993) considered the evolution of risk preferences and showed that it is better to have 4 offspring rather than 2 or 6 (because  $4^n > 2^{n/2} \times 6^{n/2}$  for  $n \in \mathbb{Z}^+$ ), thus justifying risk aversion for gains. More generally, when wealth is generated by a multiplicative process such as a financial market, it is  $\log_e(\text{wealth})$  that is additive. If one is risk neutral in terms of  $\log_e(\text{wealth})$ , because the log utility function is concave, it follows that one must exhibit a small degree of risk aversion regarding wealth. We also exhibit a preference for known risks over unknown risks, that is, we prefer known probabilities<sup>1</sup>. This is known as *ambiguity aversion*.

Research shows that risk tolerance varies across different groups of people. Barsky, *et al.* (1997) found that the highly educated, very wealthy, heavy drinkers, those without health insurance, immigrants, Jewish, Hispanics and (especially) Asians are risk seeking, whilst those with an average education, average wealth, average income, employees with health insurance and people in their sixties tend to be risk averse. Hsee and Weber (1999) found that the Chinese are more risk-seeking than Americans in the investment domain. Lau and Ranyard (2005) found that the Chinese exhibited significantly less probabilistic thinking and made riskier gambling decisions than the English.

The idea of *loss aversion* is that losses and disadvantages have a greater impact on preferences than gains and advantages. In a reversal of the cause and effect in previous hypotheses, Gal (2006) proposes that the status quo bias explains the endowment effect and the risky bet premium, and that the principle of loss aversion should be abandoned, in other words, apparent ‘loss aversion’ is actually due to a preference for the status quo. Living in groups meant that respect for private property would have likely evolved as a Nash equilibrium. Indeed, Gintis (2007) explains that private property probably evolved and shows how it gives rise to the endowment effect, and thus loss aversion.

The problem of how to maximize growth of wealth has been solved (maximize the expected value of the logarithm of wealth after each period (Kelly 1956; Breiman 1961)), but most investors are unwilling to endure the volatility of wealth that such a strategy entails. For this reason, a compromise between an optimal growth strategy and the security of holding cash has been suggested (MacLean and Ziemba 1986, 1991; MacLean, Ziemba and Blazenko 1992; MacLean and Ziemba 1999; MacLean, *et al.* 2004). In contrast, McDonnell (2008) recommends combining the use of logarithms for maximum growth of wealth (Kelly 1956) with the logarithmic utility function (Bernoulli 1738), resulting in an iterated log function,  $\log_e(1 + \log_e(1 + r))$ , where  $r$  is return.

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<sup>1</sup>As de Finetti famously noted, ‘probability does not exist’, but the point here is that we prefer to assign a subjective probability with ease.

*Expected utility theory* (also known as von-Neumann Morgenstern utility) (Bernoulli 1738; von Neumann and Morgenstern 1944; Bernoulli 1954)<sup>2</sup> states that when making decisions under risk people choose the option with the highest utility, which is dependent on the potential outcomes,  $O_i$ , the utility of each outcome,  $U(O_i)$ , and the probability of occurrence of each outcome,  $P(O_i)$ , as described by the following equation:

$$\text{Utility} = \sum_{i=1}^n U(O_i)P(O_i). \quad (1)$$

In this instance, an outcome would be final wealth.

The second-most cited paper ever to appear in *Econometrica*, the prestigious academic journal of economics, was written by the two psychologists Kahneman and Tversky (1979). They present a critique of expected utility theory as a descriptive model of decision making under risk and develop an alternative model, which they call *prospect theory*. Kahneman and Tversky found empirically that people underweight outcomes that are merely probable in comparison with outcomes that are obtained with certainty; also that people generally discard components that are shared by all prospects under consideration. Under prospect theory, value is assigned to gains and losses rather than to final assets; also probabilities are replaced by decision weights. The value function is defined on deviations from a reference point and is normally concave for gains (implying risk aversion), commonly convex for losses (risk seeking) and is generally steeper for losses than for gains (loss aversion) (see Figure 1 below). Decision weights are generally lower than the corresponding probabilities, except in the range of low probabilities (see Figure 2 (page 5)).

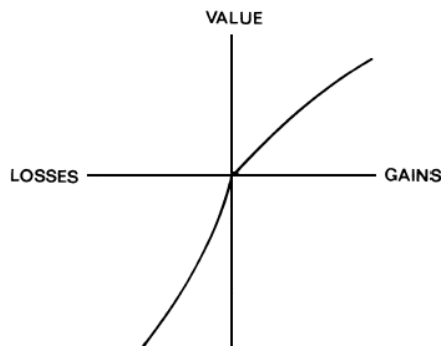


Figure 1: A hypothetical value function (Kahneman and Tversky 1979)

Tversky and Kahneman (1992) superseded their original implementation of prospect theory with *cumulative prospect theory*. The new methodology employs cumulative rather than separable decision weights, applies to uncertain as well as

<sup>2</sup>Bernoulli (1954) is a translation of Bernoulli (1738) by Louise Sommer.

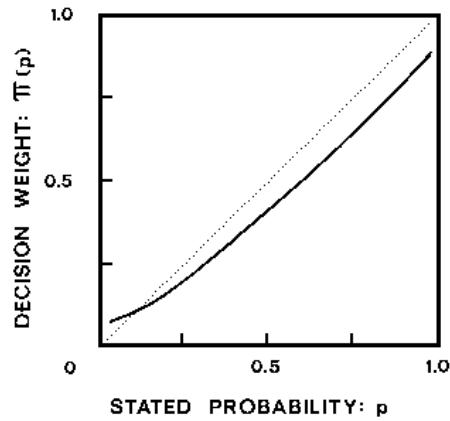


Figure 2: A hypothetical weighting function (Kahneman and Tversky 1979)

to risky prospects with any number of outcomes and it allows different weighting functions for gains and for losses (see Figure 3 below). The theory—which they confirmed by experiment—predicts a distinctive fourfold pattern of risk attitudes: risk aversion for gains and risk seeking for losses of high probability; risk seeking for gains and risk aversion for losses of low probability. Where  $p$  is

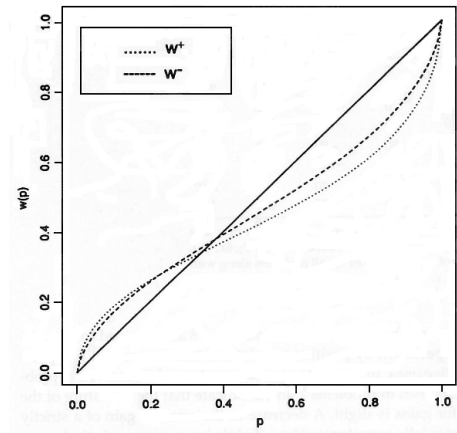


Figure 3: Weighting functions for gains ( $w^+$ ) and losses ( $w^-$ ) based on median estimates of  $\gamma$  and  $\delta$  (Tversky and Kahneman 1992)

the probability and  $\gamma$  and  $\delta$  constants, the probability weighting functions for

gains and losses respectively are:

$$w^+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}} \quad (2)$$

$$w^-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{\frac{1}{\delta}}} \quad (3)$$

The value function (where the loss aversion parameter is  $\lambda$ ) (taken from Köbberling (2002)) is as follows:

$$v(x) = \begin{cases} f(x) & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ \lambda g(x) & \text{if } x < 0 \end{cases} \quad (4)$$

where  $f(x)$  and  $g(x)$  are defined with constants  $\alpha$  and  $\beta$  as follows:

$$f(x) = \begin{cases} x^\alpha & \text{if } \alpha > 0 \\ \log(x) & \text{if } \alpha = 0 \\ 1 - (1+x)^\alpha & \text{if } \alpha < 0 \end{cases} \quad (5)$$

$$g(x) = \begin{cases} -(-x)^\beta & \text{if } \beta > 0 \\ -\log(-x) & \text{if } \beta = 0 \\ (1-x)^\beta - 1 & \text{if } \beta < 0 \end{cases} \quad (6)$$

Tversky and Kahneman (1992)'s empirical work resulted in the following median values:  $\alpha = 0.88$ ,  $\beta = 0.88$ ,  $\lambda = 2.25$ ,  $\gamma = 0.61$  and  $\delta = 0.69$ .

I have developed a cumulative prospect theory calculator, which is available online.<sup>3</sup> Also, cumulative prospect theory's certainty equivalent makes up part of a performance measurement calculator which I wrote for the Web and Excel.<sup>4</sup>

Note that there are two fundamental reasons why prospect theory (which calculates value) is inconsistent with expected utility theory. Firstly, whilst utility is necessarily linear in the probabilities, value is not. Secondly, whereas utility is dependent on final wealth, value is defined in terms of gains and losses (deviations from current wealth).

More recent developments have improved upon cumulative prospect theory, such as the transfer of attention exchange model (Birnbaum 2008). Whilst Harrison and Rutström (2009) propose a reconciliation of expected utility theory and prospect theory.<sup>5</sup>

The *Sharpe ratio* (Sharpe 1994) is the most prevalent performance metric in use by the financial industry. Where  $r_p$  is the asset or portfolio return,  $r_f$  is the return on a benchmark asset, such as the risk free rate of return,  $E[r_p - r_f]$  is the expected value of the excess of the portfolio return over the benchmark return, and  $\sigma = \sqrt{\text{Var}[r_p - r_f]}$  is the standard deviation of the excess return,

$$\text{Sharpe ratio} = \frac{E[r_p - r_f]}{\sigma}. \quad (7)$$

<sup>3</sup><http://prospect-theory.behaviouralfinance.net/cpt-calculator.php>

<sup>4</sup><http://www.performance-measurement.org>

<sup>5</sup>Thanks to Donald A. Hantula for drawing my attention to these two articles.

The Sharpe ratio makes implicit assumptions which stem from the capital asset pricing model (CAPM) (Treyner 1962; Sharpe 1964; Lintner 1965; Mossin 1966)<sup>6</sup>: it assumes either 1) normally distributed returns *or* 2) mean-variance preferences. Both assumptions are suspect:

1. The returns generated by most hedge funds exhibit negative skewness (Kat and Lu 2002).
2. In addition to the mean and variance, people also care about skewness (they like it positive) and kurtosis (they don't like it), and higher moments matter too (Scott and Horvath 1980). Hakansson and Ziemba (1995) point out that 'in solving for the growth-optimal strategy, *all* of the moments of the return distributions matter, with positive skewness being particularly favoured'.

Because the Sharpe ratio is oblivious of all moments higher than the variance, it is prone to manipulation. Goetzmann, *et al.* (2002) proved that an optimal (high) Sharpe ratio strategy would produce a distribution with a truncated right tail and a fat left tail, as shown in Figure 4 below. That is, with the expected return being held constant, generate regular modest profits punctuated by occasional crashes, i.e. negative skewness. As mentioned in 2. above, most investors prefer positive skewness, therefore, although a high Sharpe ratio is good thing, a high Sharpe ratio strategy is a bad thing.

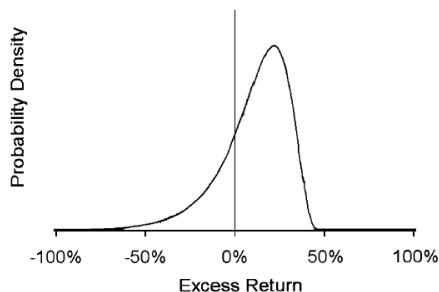


Figure 4: Maximal Sharpe ratio (Goetzmann, *et al.* 2002)

## 4 Methodology

The goal is to produce a performance metric that is a hybrid of the normative expected utility theory and the descriptive prospect theory. The problem with using expected utility theory alone as a prescriptive model is that there is no

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<sup>6</sup>Under CAPM, the portfolio on the efficient frontier with the highest Sharpe ratio is the market portfolio. The slope of the capital market line equals the market (i.e. index) Sharpe ratio.

principled way of quantifying risk. Although the model generally assumes that individuals are risk averse, it does not dictate to what degree. The problem with using prospect theory alone is that although people are generally risk averse for gains, they tend to be risk seeking for losses. Such preferences, such as purchasing lottery tickets, are detrimental to one's wealth. The solution is to base our desired prescriptive model, as far as possible, on the more rational model, namely expected utility theory, and then employ prospect theory to determine risk aversion. In other words, we must ensure that utility is linear in the probabilities (so as not to violate expected utility theory), and consider both changes in wealth (for prospect theory) and final wealth (for expected utility theory).

There are two potential avenues, both of which shall be considered. One is to make a generic one-size-fits-all risk metric by employing cumulative prospect theory with the default parameters found empirically by Tversky and Kahneman (1992) to determine the utility curve. The benefits of such an approach are that the performance of investments, funds or fund managers may be universally compared, and figures quoted.

The second approach is to accept that risk is a personal thing and consider individuals' personal risk profiles. One could get many subjects to fill out questionnaires or partake in experiments consisting of investment choices, where the options are constrained by the axioms of expected utility theory, and select parameters on an individual basis. This second approach is more flexible.

If risk profiles vary from person to person, do they also vary across time? As, ultimately, men are competing with other men, and women with other women, it is our wealth relative to others that counts. In that sense, it seems reasonable that our risk tolerances should also be dependent on the wealth and risk preferences of those we are competing with, so may vary over time.

Regarding the question of calculating an aggregate risk profile despite accepting that risk is a personal thing, for dealing with matters such as the tragedy of the global commons, global warming, the first approach above seems reasonable.

The value of money in the future is determined by interest rates. But how do we determine the social discount rate? This is a moral judgement, and morality is a product of the gene-centred evolutionary forces which shape human social psychology. Although our ultimate motivation is gene propagation (long term), our proximate motivation is sex and to care for our kin (short term). In other words, we have evolved on a need-to-know basis, and we care only about our living relatives (the immediate future). A minority of people deceive themselves into helping to solve our ultimate goal on a global basis, enabling propagation of their genes and everyone else's (aka 'saving the world'), because this helps them achieve their proximate goal, via their enhanced status, of sex.

## 5 Dissemination

The research findings should percolate both academia and the financial industry. To help facilitate this, the questionnaire for determining personal risk profiles



could be web-based, allowing the concept to rapidly propagate widely. The work should generalize beyond the domain of finance and make an interesting contribution to decision making under risk in general. Also, the consideration of risk on both a universal and temporal basis should extend the potential audience to those with an interest in global and future (yet pressing) matters such as climate change.

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